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Aposyndesis and cut points which are related to refinable maps

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1. Results. We shall fix a refinable map  $r : X \rightarrow Y$  between continua (compact connected metric spaces). Let  $y$  be a point of  $Y$ . There is a  $(1/n)$ -refinement  $r_n$  of  $r$  for each positive integer  $n$  such that  $\{r_n^{-1}(y)\}$  converges to some point  $\hat{y}$  of  $X$  (see [1]).

Theorem 1. If  $X$  is aposyndetic (resp. semi-aposyndetic, mutually aposyndetic, semi-locally connected, locally remotely connected) at  $\hat{y}$ , then so is  $Y$  at  $y$ .

A closed subset  $F$  of a space  $M$  is said to separate (resp. weakly separate)  $M$  if  $M - F$  is not connected (not continuumwise connected). When  $F$  consists of only one point,  $F = \{p\}$ , then  $p$  is said to be a cut point (a weak cut point) of  $M$  provided that  $\{p\}$  separates (weakly separates)  $M$ .

Theorem 2. (i) A point  $y$  of  $Y$  is a weak cut point of  $Y$  if and only if  $r^{-1}(y)$  weakly separates  $X$ .

(ii) If  $Y$  is semi-locally connected at  $y$ , then  $y$  is a cut point of  $Y$  if and only if  $r^{-1}(y)$  separates  $X$ .

2. Proof of Theorem 1. This Theorem is a pointwise version of the results in [2] except for the last case.

First let us assume that  $X$  is mutually aposyndetic at  $\hat{y}$ .

Let  $z$  be a point of  $Y - \{y\}$ . We may assume that  $\{r_n^{-1}(z)\}$  converges to some point  $\hat{z}$  of  $X$ . Since  $\hat{y} \neq \hat{z}$ , there are disjoint continuum neighborhoods  $H$  and  $K$  of  $\hat{y}$  and  $\hat{z}$  in  $X$  respectively. Choose an integer  $n$  so that  $r_n^{-1}(y) \subset \text{int}(H)$  and  $r_n^{-1}(z) \subset \text{int}(K)$ . Then  $r_n(H)$  and  $r_n(K)$  are disjoint continuum neighborhoods of  $y$  and  $z$  in  $Y$  respectively. Therefore  $Y$  is mutually aposyndetic at  $y$ .

Second let us assume that  $X$  is locally remotely connected at  $\hat{y}$  (i.e. each neighborhood of  $\hat{y}$  contains an open neighborhood of  $\hat{y}$  whose complement is connected). Let  $U$  be a given neighborhood of  $y$  in  $Y$  and let  $V$  be an open neighborhood of  $y$  such that  $\bar{V} \subset U$ . Since  $r^{-1}(V)$  is a neighborhood of  $\hat{y}$ , there is an open neighborhood  $V_0$  of  $\hat{y}$  in  $r^{-1}(V)$  such that  $X - V_0$  is connected. Choose an integer  $n$  so large that it satisfies  $r_n^{-1}(y) \subset V_0$  and  $1/n < d(Y - U, \bar{V})$ . Then  $U_0 = Y - r_n(X - V_0)$  is an open neighborhood of  $y$  in  $U$  such that  $Y - U_0$  is connected.

The remaining cases can be proved by slight modifications of the proof of the case of mutual aposyndesis.

3. Proof of Theorem 2. (i). Let us assume that  $y$  is not a weak cut point of  $Y$ . Then there is a sequence  $\{K_m\}$  of subcontinua of  $Y$  such that  $K_1 \subset K_2 \subset \dots$ , and  $Y - \{y\} = \bigcup_{m=1}^{\infty} K_m$ . Inductively we can choose a sequence of subsequences  $\{r_{m,n}\}_{n=1}^{\infty}$ ,  $m = 1, 2, \dots$ , of  $\{r_n\}_{n=1}^{\infty}$  such that

$$(1) \quad \{r_{m+1,n}\}_{n=1}^{\infty} \text{ is a subsequence of } \{r_{m,n}\}_{n=1}^{\infty},$$

(2)  $\{r_{m,n}^{-1}(K_m)\}_{n=1}^{\infty}$  converges to some continuum  $\hat{K}_m$  for  $m = 1, 2, \dots$ .

The sequence  $\{\hat{K}_m\}_{m=1}^{\infty}$  is increasing and satisfies  $X - r^{-1}(y) = \bigcup_{m=1}^{\infty} K_m$ . Hence  $r^{-1}(y)$  does not weakly separate  $X$ .

(ii). Assume that  $y$  is not a cut point of  $Y$ . Since  $Y$  is semi-locally connected at  $y$ , Whyburn's Theorem implies that  $y$  is not a weak cut point. Hence  $Y$  is locally remotely connected at  $y$ . Suppose that  $r^{-1}(y)$  separates  $a$  from  $b$  in  $X$ . There are open neighborhoods  $V_1$  and  $V_2$  of  $y$  such that  $\bar{V}_2 \subset V_1$ ,  $\bar{V}_1 \subset Y - \{r(a), r(b)\}$ , and that  $Y - V_1$  is connected. We may assume that  $\{r_n^{-1}(\bar{V}_1)\}$  and  $\{r_n^{-1}(\bar{V}_2)\}$  converge to some closed sets  $K_1$  and  $K_2$  respectively. Since  $\text{int}(K_2)$  separates  $a$  from  $b$  in  $X$ , there is a separation  $X - \text{int}(K_2) = A \cup B$ , where  $a \in A$  and  $b \in B$ . Choose an integer  $n$  so that  $1/n < \min\{d(A, B), d(\bar{V}_2, Y - V_1), d(r(a), \bar{V}_1), d(r(b), \bar{V}_1)\}$ . It is easy to see that  $Y - V_1 = (r_n(A) - V_1) \cup (r_n(B) - V_1)$  is a separation of  $Y - V_1$ . This contradiction implies that  $r^{-1}(y)$  does not separate  $X$ .

The "only if" parts are easy to prove.

#### References

- [1] J. Ford and J. W. Rogers, Refinable maps, Colloq. Math., 39 (1978), 263-269.
- [2] H. Hosokawa, Aposyndesis and coherence of continua under refinable maps, to appear in Tsukuba J. Math., 5 (1983).